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The Interplay Between Skew Braces and Hopf-Galois Theory

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Abstracts

A Family of Simple Skew Braces

Nigel Byott
Univeristy of Exeter

Many examples of simple braces have been constructed, but few examples are known of simple skew braces with nonabelian additive group. In this talk, I will sketch a proof that, for any primes p, q such that q divides $(p^p - 1)/(p - 1)$, there are exactly two simple skew braces of order $n = p^p q$ (up to isomorphism). On the one hand, this provides an infinite family of simple skew braces which are not braces and which do not arise from nonabelian simple groups. On the other hand, it gives a classification of simple skew braces of order n when n has the above form.

Cabling for non-involutive solutions

Ilaria Colazzo
Univeristy of Exeter

This talk is based on joint work with Arne Van Antwerpen. We will focus on bijective non-degenerate solutions to the Yang-Baxter equation (YBE). We will introduce the cabling for bijective non-degenerate solutions and show that this is a useful tool for dealing with indecomposability.

Homotopy-theoretical constructions of biracks, welded-biracks, and generalisations

João Faria Martins
University of Leeds

I will report on recent work on homotopy-theoretical constructions of biracks, welded biracks, and how they provide representations of the loop braid group. Time permitting, I will also address a minimal categorification of biracks, which provides a generalisation of the R-matrix of the quantum double of a finite group, and similarly yields representations of the loop braid group.

Joint work with: Paul Martin, Alex Bulivant, and Paul Martin and Celeste Damiani, and also based on old joint work with with Louis Kauffman.

Algebraic structure of Yang-Baxter algebras

Eric Jespers

Vrije Universiteit Brussel

Given a set X and a field K, one associates to a map $r : X \times X \to X \times X$, with $(x, y) \mapsto (\lambda_x(y), \rho_y(x))$, a quadratic algebra $\mathcal{A}_K(X, r) = K\langle X \mid xy - \lambda_x(y)\rho_y(x),\ x, y \in X\rangle$. This algebra often is called the structure algebra of (X, r). It has a presentation defined by quadratic word relations and thus it is the monoid algebra of the structure monoid $M(X, r)$, i.e. the monoid generated by the set X and subject to the "same" word relations. This algebra is the associative ring theoretic tool to investigate the map r and has attracted a lot of attention in case r satisfies the braided relation, i.e. (X, r) is a set-theoretic solution of the Yang-Baxter equation. Fundamental results have been proven for algebras of such solutions. For example, if X is finite and r is bijective and non-degenerate, i.e. all maps λ_x and ρ_y are bijective, then the algebra satisfies a polynomial identity and is left and right Noetherian and has finite Gelfand-Kirillov dimension. If, furthermore, r is involutive then these algebras share many properties with polynomial algebras in commuting variables.

The aim of this lecture is to explain the intriguing relationship between the algebraic structure of the structure algebras $\mathcal{A}_K(X, r)$ and the finite left non-degenerate (i.e. all maps λ_x are bijective and X is finite) set-theoretic solutions (X, r) of the Yang-Baxter equation. The main focus is on when such algebras are Noetherian, prime, semiprime, representable and the Gelfand-Kirillov dimension.

Skew Bracoids in the Holomorph

Isabel Martin-Lyon

Univeristy of Keele

The skew bracoid is a generalisation of the skew brace corresponding to Hopf-Galois structures on separable, but not necessarily normal, extensions of fields. We briefly review this correspondence via transitive subgroups of the holomorph, accounting for the slight discrepancy in the established routes. We then translate some structural properties of skew bracoids to their holomorph formulation, giving a new perspective on their corresponding Hopf-Galois structures, while paying particular attention to questions of Hopf-Galois correspondence.

Indecomposable solutions of the Yang–Baxter equation, generators of skew braces and Hopf–Galois structures

Senne Trappeniers
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Left braces, and later skew left braces, were introduced as a tool to study and produce solutions of the set-theoretic Yang–Baxter equation. Ever since then, this connection remains one of the driving forces behind research on skew braces. However, also its relation with Hopf–Galois structures starts to gain more attraction.

In this talk, based on a joint work with M. Castelli and a joint work with L. Stefanello, we first take a look at how a solution of the Yang–Baxter equation being indecomposable relates to its permutation skew brace being one-generated in a certain sense. In order to stay true to the name of this meeting, we also shortly look at how skew braces interact with Hopf–Galois structures and we try to link the earlier results on generating sets of a skew brace to statements about its related Hopf–Galois structures.

Skew bracoids and the Yang-Baxter equation

Paul Truman
Univeristy of Keele

Skew bracoids can be used to study Hopf–Galois structures on field extensions that are separable but not necessarily normal. It is natural to ask whether they have any connection with the (set theoretic) Yang-Baxter equation. In joint work with I. Colazzo and I. Martin-Lyons we have shown that certain skew bracoids yield left degenerate solutions of the YBE. Here we present new examples of skew bracoids, which also yield left degenerate solutions of the YBE. Our construction is neither a generalization nor a specialization of the previous one, which leads us to hope that both are part of a broader picture. Joint work with Alan Koch.

The non-fixing graph of a skew brace

Arne Van Antwerpen
Vrije Universiteit Brussel

In this talk, based on joint work with Silvia Properzi, we recall some core concepts of skew braces. Next, we introduce the non-fixing graph of a skew brace, inspired by the conjugacy class graph as defined by Bertram, Herzog and Mann. This graph measures the size of λ-orbits of a skew brace. We investigate some general properties of this graph and classify extremal cases. In particular, we classify the skew braces with a 1 vertex graph and those with a 2 vertex graph. If time allows, we discuss a straightforward extension of this graph and several other graphs related to set-theoretic solutions and skew braces.
