The Interplay Between Skew Braces and Hopf-Galois Theory

Vrije Universiteit Brussel (Belgium)

20th January, 2025

Schedule

(09:30-10:30) **Andrea Caranti**: *Finite p-groups of class two with a large/small multiple holomorph* joint work with Cindy (SIn Yi) Tsang

(10:30-11:00) Coffee break

(11:00-12.00) **Marino Gran**: *Semi-abelian categories, cocommutative Hopf algebras and Hopf braces*

(12:00-12:30) **Davide Ferri**: *Matched pairs and Yetter-Drinfeld braces*

(12:30-14:00) Lunch

(14:30-15:30) **Paul Truman**: *Two sided bracoids, holomorphs, centralisers, and all that*

(15:30-16:00) Coffee break

(16:00-16:30) **Charlotte Roelants**: *Killing Forms on Finite Groups*

(16:30-17:30) **Paolo Saracco**: *Trusses, heaps of modules, and set-theoretic solutions to the Yang-Baxter equation*

Abstracts

Finite p-groups of class two with a large/small multiple holomorph

20 Jan
09:30
10:30

Andrea Caranti

(joint work with Cindy (SIn Yi) Tsang)
Università di Trento (Italy)

Cindy Tsang and I have constructed examples of finite p-groups of class two with large/small multiple holomorphs. In this talk I would like to discuss the techniques we have used for these purposes. These tools, which we have learned from Hermann Heineken, basically belong to linear algebra.

Semi-abelian categories, cocommutative Hopf algebras and Hopf braces

20 Jan
10:00
12:00

Marino Gran

Université catholique de Louvain (Belgium)

In this talk I'll present some recent results on the interactions between Hopf algebra theory and semi-abelian categories. Semi-abelian categories [1] have played a central role in categorical algebra during the last 25 years. After recalling the motivation for studying them, some basic concepts and a few examples, we shall explain why the category of cocommutative Hopf algebras is semi-abelian [2]. We shall then turn our attention to (cocommutative) Hopf braces [3], that extend cocommutative Hopf algebras and can be seen as a Hopf-theoretic generalization of skew braces [4], that are useful to study solutions of the Yang-Baxter equation. In the recent article with Andrea Sciandra [5] we have investigated the exactness properties of the category of Hopf braces, and some natural constructions therein. First, we show that cocommutative Hopf braces form a semi-abelian category, that is also strongly protomodular. When the base field is algebraically closed and has zero characteristic one can find an interesting torsion theory therein, whose torsion-free subcategory is equivalent to the variety of skew braces, which turns out to be also a localization. Finally, we provide some explicit descriptions of the categorical commutator and of the central extensions of Hopf braces, that are likely to be useful for some new applications in non-abelian (co)homology theory.

References

[1] G. Janelidze, L. Marki and W. Tholen, Semi-abelian categories, J. Pure Appl. Algebra 168 (2002) 367-386

[2] M. Gran, F. Sterck and J. Vercruysse, A semi-abelian extension of a theorem by Takeuchi, J. Pure Appl. Algebra 223 (2019) 4171-4190

[3] I. Angiono, C. Galindo, L. Vendramin, Hopf braces and Yang-Baxter operators, Proc. Amer. Math. Soc. 145 (2017) 1981-1995

[4] L. Guarnieri, L. Vendramin, Skew braces and the Yang-Baxter equation. Math. Comp. 86 (2017) 2519-2534

[5] M. Gran and A. Sciandra, Hopf braces and semi-abelian categories, preprint, arXiv:2411.19238 (2024)

---

20 Jan
12:00
12:30

Matched pairs and Yetter-Drinfeld braces

Davide Ferri

(joint work with A. Sciandra)
Università di Torino (Italy) and Vrije Universiteit Brussel (Belgium)

It is known that matched pairs of actions on a group are equivalent to skew braces. This has a natural Hopf-theoretic version: matched pairs of actions on a cocommutative Hopf algebra are equivalent to cocommutative Hopf braces. However, this is false without cocommutativity, thus suggesting that Hopf braces are the wrong object to be used for this correspondence. Here we prove that matched pairs of actions on a Hopf algebra, in the most general case, correspond to novel structures called Yetter-Drinfeld braces. We observe that Yetter-Drinfeld braces generalise cocommutative Hopf braces, but they also arise naturally from every coquasitriangular Hopf algebra, and they provide an interpretation for Majid's transmutation.

---

20 Jan
14:30
15:30

Two sided bracoids, holomorphs, centralisers, and all that

Paul Truman

University of Keele (UK)

Skew bracoids (in the sense of Martin-Lyons and the speaker) are generalisations of skew braces with connections to Hopf-Galois theory, semibraces, and set-theoretic Yang-Baxter equation. It is natural to investigate which interesting definitions and results concerning skew braces have skew bracoid counterparts. Malinowska has defined two-sided bracoids and used them to generalise results of Rump concerning the connection between radical rings and two-sided braces. In this talk we summarise Malinowska's work and suggest that her definition of two-sided bracoids might be more restrictive than necessary. In searching for an alternative definition we find motivation for generalising the definition of a skew bracoid even further.

Killing Forms on Finite Groups

20 Jan
16:00
16:30

Charlotte Roelants

Vrije Universiteit Brussel (Belgium)

Killing forms are bilinear forms most well-known in the context of Lie algebras. Applying them to certain Lie algebras based on finite groups yields an expression in terms of centralizers in the group. In this talk, based on a collaboration with Kevin Piterman, we discuss some problems concerning the non-degeneracy and irreducibility of these forms. We study the case of the finite simple groups $\mathrm{PSL}(2, q)$ and cover some recent results on involutions in simple groups of Lie type and Lie rank 1.

Trusses, heaps of modules, and set-theoretic solutions to the Yang-Baxter equation

20 Jan
16:30
17:30

Paolo Saracco

ULB - Université libre de Bruxelles (Belgium)

This would like to be a friendly introduction to the theory of trusses, their heaps of modules and their connections with affine modules, with a jaunt into spindles, quandles and (non-degenerate) solutions to the set-theoretic Yang-Baxter equation that naturally arise from them.

(based on a joint project with S. Breaz, T. Brzeziński and B. Rybołowicz)
